Phys 410
Fall 2013
Lecture #7 Summary
24 September, 2013

One-dimensional problems, although they appear to be artificial, pop up frequently in the solution of three-dimensional problems. So far we have no mention of time in the evolution. We can find $x(t)$ starting with the mechanical energy, and at least one additional piece of information (the sign of $\dot{x}$). We showed that the statement of mechanical energy conservation can be re-written as $t=\sqrt{\frac{m}{2}}\int_{x_0}^{x}\frac{dx'}{\sqrt{E-U(x')}}$. This can be solved for $x(t)$, and from that one can determine the velocity, acceleration, etc. as functions of time. We did problem 4.28 from HW 3 and solved the above equation for $x(t)$ for a simple harmonic oscillator.

We considered energy for motion in curvilinear one-dimensional systems. An example is a car moving on a roller coaster track. Consider a particle confined to move along a one-dimensional 'track' parameterized by its displacement s from some arbitrary origin. It has a kinetic energy $T=\frac{1}{2}m\dot{s}^2$. The kinetic energy can be altered by applying a tangential force and doing work on the particle. Newton's second law can be stated as $m\ddot{s}=F_{net}$. If the tangential force is conservative, then you can define a potential energy U, and a total mechanical energy $E=T+U$.

We next considered central forces. These are forces that are everywhere directed toward a fixed force center. Such a force has the form $\vec{F}(\vec{r})=f(\vec{r})\hat{r}$. If further the force is spherically symmetric, then the scalar function depends only on the radial distance and not the angular coordinates: $f(\vec{r})=f(r)$.

There are two statements that can be made about central forces:

1) A central force that is conservative is automatically spherically symmetric,
2) A central force that is spherically symmetric is automatically conservative.

We proved the first of these two statements. If the force is conservative, then it can be represented in terms of the gradient of a scalar potential: $\vec{F}=-\vec{\nabla}U(\vec{r})$. Using the gradient in spherical coordinates, derived in class ($\vec{\nabla}=\hat{r}\partial/\partial r+(\hat{\theta}/r)\partial/\partial\theta+(\hat{\phi}/r\sin\theta)\partial/\partial\phi$), we find that a central force (dependent on $\hat{r}$ only) requires that $\partial U/\partial\theta=\partial U/\partial\phi=0$. This means that the potential energy depends only on the radial coordinate: $U=U(r)$. In turn, the central force can only depend on the scalar radial coordinate: $\vec{F}=-\hat{r}\,\partial U(r)/\partial r$, which means that it is spherically symmetric (i.e. no dependence of the potential and force on the angular coordinates θ,ϕ). The one-dimensional nature of the potential energy and force will have benefits later when we look at the two-body problem.

We considered two-particle conservative-force interactions and found that the potential function $U(\vec{r}_1 - \vec{r}_2)$ does double duty by representing two forces: $\vec{F}_{12}$ and $\vec{F}_{21}$. In other words, one can recover the forces of interaction between the two particles as $\vec{F}_{12} = -\vec{\nabla}_1 U(\vec{r}_1 - \vec{r}_2)$ and $\vec{F}_{21} = -\vec{\nabla}_2 U(\vec{r}_1 - \vec{r}_2)$, where the subscripts on the gradients denote derivatives with respect to the coordinates of one of the particles. In addition, this single potential energy function properly keeps track of the total potential energy of the two particles interacting by means of the associated conservative forces. The total mechanical energy of the two-particle system is given by $E = T_1 + T_2 + U(\vec{r}_1 - \vec{r}_2)$, and this quantity is conserved. This treatment can be generalized to an arbitrary number of particles interacting with each other by means of conservative forces, and under the influence of external conservative forces.